

Thermoeconomic optimization and parametric study of an irreversible Stirling heat pump cycle

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Abstract

The thermo-economic optimization of an irreversible Stirling heat pump cycle with a detail parametric study for the finite heat capacity of external reservoirs is presented in this article. The external irreversibility is due to finite temperature difference between the working fluid and the external reservoirs while the internal irreversibility is due to the regenerative heat loss. The Thermo-economic function is defined as the heating load divided by the total cost of the system along with the running cost. The Thermo-economic function is optimized with respect to the working fluid temperatures and the values for various parameters at the optimal operating condition are calculated. The effects of different operating parameters on the performance of the cycle have been studied. It is found that the effect of regenerative effectiveness and the economic parameter are more pronounced than that of the other parameters.

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1. Introduction

The Stirling cycle is one of the important models of refrigeration systems for the production of desirable temperatures and the new developments in the design were proposed after the new concept of finite time thermodynamics [1,2] came into existence. It was remarked that the efficiency of an engine operating at maximum power is given by a different formula [$\eta_m = 1 - \sqrt{T_L/T_H}$], which is always smaller than the well known Carnot formula [$\eta_C = 1 - T_L/T_H$], where T_L and T_H are, respectively, the temperatures of sink and source for heat engines while it is reverse for heat pump cycles] and agrees much better with the measured efficiencies of operating installations. Leff and Teeters [3] have noted that the straight forwards C-A calculations will not work for a reversed Carnot cycle because there is no “Natural Maximum”. Blanchard [4] has applied the Lagrangian method of undetermined multiplier to find out the COP of an endoreversible Carnot heat pump operated at minimum power input for a given heating load.

In recent years the performance of the different heat engines and refrigeration systems have been investigated by a number of researchers using the concept of finite time thermodynamics [5–15], while others have applied the ecological [16–23] and thermo-economic [24–29] approach for various operating conditions.

In this paper we will investigate the thermo-economic optimization along with a detail parametric study of an irreversible Stirling heat pump cycle for different operating conditions using the concept of finite time thermodynamics.

2. System description

It is well known that the working substance of the Stirling cycle may be a gas, a magnetic material etc. and for different working fluids, this cycle has different performance characteristics. When the working substance of the cycle is the perfect/ideal gas the Stirling cycle consists of two isothermal and two isochoric processes. The schematic and T-S diagrams are shown in Fig. 1. This cycle approximates the expansion stroke of a real cycle as an isothermal process 1–2 with an irreversible isothermal heat addition at temperature T_c from a heat source of finite heat capacity

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Nomenclature

A	area..... m^2
C	heat capacitance rate..... $\text{kW}\cdot\text{K}^{-1}$
c_f	molar specific heat..... $\text{kJ}\cdot\text{kmol}^{-1}\cdot\text{K}^{-1}$
COP	coefficient of performance
P	power..... kW
Q	heat..... kJ
R	heating load..... kW
R_0	gas constant..... $\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$
T	temperature..... K
t	time..... s
U	overall heat transfer coefficient.. $\text{kW}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$
1, 2, 3, 4	state points

Greek letters

ε	effectiveness
λ	volume ratio

Subscripts

c	cold side/source side
f	fluid
h, H	sink side/heating
L	heat source
max /m	maximum/optimum condition
R	regenerator

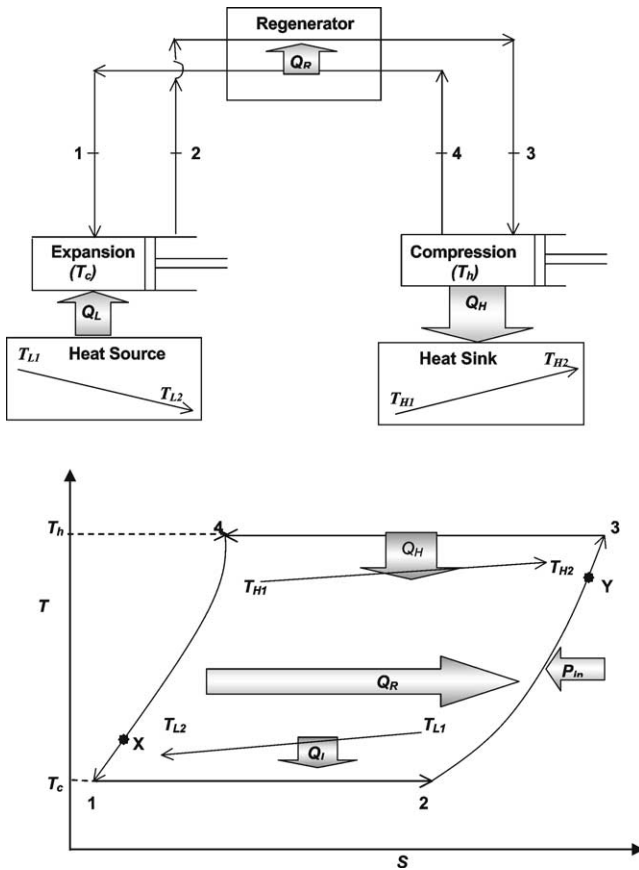


Fig. 1. Schematic and T-S diagrams of an Irreversible Stirling Heat Pump.

whose temperature varies from T_{L1} to T_{L2} . The heat addition to the working fluid from the regenerator is modeled as an isochoric process 2–3. The compression stroke is modeled as an isothermal process 3–4 with an irreversible heat rejection at temperature T_h to the heat sink of finite heat capacity whose temperature varies from T_{H1} to T_{H2} . Finally, the heat rejection from the working fluid to the regenerator is modeled as an isochoric process 4–1, thereby completing the cycle.

As mentioned earlier, the heat transfer processes 1–2 and 3–4 in a real cycle must occur in finite time. This requires that these heat processes must be proceeded through a finite temperature difference and therefore, defined as being externally irreversible. There is also some heat loss through the regenerator, as an ideal regeneration requires an infinite regeneration time or area, which is not the case in practice.

3. Thermodynamic analysis

Let Q_c is the amount of heat absorbed from the source at temperature T_c and Q_h is the amount of heat released to the sink at temperature T_h during two isothermal processes, then:

$$Q_h = T_h \Delta S = C_H (T_{H2} - T_{H1}) t_H = C_H \varepsilon_H (T_h - T_{H1}) t_H \quad (1)$$

$$Q_c = T_c \Delta S = C_L (T_{L1} - T_{L2}) t_L = C_L \varepsilon_L (T_{L1} - T_c) t_L \quad (2)$$

where

$$\Delta S = n R_0 \ln \lambda \quad (3)$$

λ is the volume ratio, n is the number of mole for the working fluid, and R_0 is the universal gas constant. C_H , C_L and t_H , t_L are the heat capacitance rates of sink/source reservoirs and heat rejection/addition times and ε_H and ε_L are the effectiveness of the heat exchangers for hot- and cold-side, respectively. Also from the heat transfer theory the heat transfers Q_h and Q_c will be proportional to the Log Mean Temperature Difference (LMTD), i.e.,

$$Q_h = U_H A_H (\text{LMTD})_H t_H \quad (4)$$

$$Q_c = U_L A_L (\text{LMTD})_L t_L \quad (5)$$

where $U_H A_H$ and $U_L A_L$ are the overall heat transfer coefficient-area products and $(\text{LMTD})_H$ and $(\text{LMTD})_L$ are the Log Mean Temperature Difference on sink and source side, respectively, and defined as:

$$(\text{LMTD})_H = \left[\frac{(T_h - T_{H1}) - (T_h - T_{H2})}{\ln \left(\frac{T_h - T_{H1}}{T_h - T_{H2}} \right)} \right] \quad (6a)$$

$$(\text{LMTD})_L = \left[\frac{(T_{L1} - T_c) - (T_{L2} - T_c)}{\ln\left(\frac{T_{L1} - T_c}{T_{L2} - T_c}\right)} \right] \quad (6b)$$

From Eqs. (1)–(6), the effectiveness of the hot- and cold-side heat exchangers are obtained as:

$$\varepsilon_H = 1 - e^{-N_H}, \quad \varepsilon_L = 1 - e^{-N_L} \quad (7a)$$

where N_H and N_L are the number of heat transfer units based on minimum thermal capacitance rates and are given as:

$$\left(N_H = \frac{U_H A_H}{C_H}, N_L = \frac{U_L A_L}{C_L} \right)$$

As these cycles, in general, do not possess the condition of perfect regeneration, so it is assumed reasonably that the regenerative loss per cycle is proportional to the temperature difference of the two isothermal processes as assumed by earlier workers [7–10,12–15].

$$\Delta Q_R = n c_f (1 - \varepsilon_R) (T_h - T_c) \quad (7b)$$

where c_f is the molar specific heat capacity of the working fluid and ε_R is the effectiveness of the regenerator, which is defined as:

$$\begin{aligned} \varepsilon_R &= \frac{Q_{\text{regen, actual}}}{Q_{\text{regen, ideal}}} = \frac{Q_{4X}}{Q_{41}} = \frac{Q_{2Y}}{Q_{23}} = \frac{T_h - T_X}{T_h - T_c} \\ &= \frac{T_Y - T_c}{T_h - T_c} = \frac{N_R}{1 + N_R} \end{aligned} \quad (8a)$$

where N_R is the number of transfer unit of the regenerator and defined as:

$$N_R = \frac{(U A)_R}{C_f} \quad (8b)$$

here C_f is the heat capacitance rate of the working fluid.

Owing to the influence of irreversibility of finite heat transfer, the regenerative time should be finite as compared to the two isothermal processes and is given by [5,7–10,12–15]:

$$t_R = t_3 + t_4 = 2\alpha(T_h - T_c) \quad (9)$$

where α is the proportionality constant, which is independent of the temperature difference but depends on the property of the regenerative material. Thus, the total cycle time t_{cycle} is:

$$t_{\text{cycle}} = (t_H + t_L + t_R) \quad (10)$$

When the irreversibilities mentioned above are taken into account, the net amounts of heat released to the sink and absorbed from the source are:

$$Q_H = Q_h - \Delta Q_R \quad (11)$$

$$Q_L = Q_c - \Delta Q_R \quad (12)$$

It is well known that the power input, heating load and coefficient of performance (COP) are the important parameters of the heat pump. Using the above equations we have:

$$P = \frac{(Q_H - Q_L)}{t_{\text{cycle}}} = \frac{(Q_h - Q_c)}{(t_H + t_L + t_R)} \quad (13)$$

$$R_H = \frac{Q_H}{t_{\text{cycle}}} = \frac{Q_h}{(t_H + t_L + t_R)} \quad (14)$$

$$\text{COP}_H = \frac{R_H}{P} = \frac{Q_h}{(Q_h - Q_c)} \quad (15)$$

Using Eqs. (11)–(16) we obtain:

$$\frac{Q_c}{T_c} - \frac{Q_h}{T_h} = 0 \quad (16)$$

Thus, from Eqs. (11)–(16), we have:

$$P = \frac{(x - 1)}{\left[\frac{x}{k_H(x y - T_{H1})} + \frac{1}{k_L(T_{L1} - y)} + b_1(x - 1) \right]} \quad (17)$$

$$R_H = \frac{[x - a_1(x - 1)]}{\left[\frac{x}{k_H(x y - T_{H1})} + \frac{1}{k_L(T_{L1} - y)} + b_1(x - 1) \right]} \quad (18)$$

$$\text{COP}_H = \frac{[x - a_1(x - 1)]}{(x - 1)} \quad (19)$$

where $k_H = \varepsilon_H C_H$, $k_L = \varepsilon_L C_L$, $x = T_h/T_c$, $y = T_c$, $b_1 = 2\alpha/(n R_0 \ln \lambda)$ and $a_1 = c_f(1 - \varepsilon_R)/R_0 \ln \lambda$.

The objective function of thermo-economic optimization proposed by earlier workers [24,25] is given by:

$$F = \frac{\dot{Q}_H}{C_i + C_e} \quad (20)$$

where C_i and C_e refer to annual investment and energy consumption costs.

The investment cost was considered the costs of the main system components that are the heat exchangers and the compression and expansion devices together. The investment cost of the heat exchangers is assumed to be proportional to the total heat transfer area [24,25]. On the other hand, the investment cost due to the compression and expansion devices is assumed to be proportional to their compression/expansion capacities or the required power input. Thus the annual investment cost of the system can be given by:

$$\begin{aligned} C_i &= a(A_H + A_L + A_R) + b_2 P \\ &= a(A_H + A_L + A_R) + b_2(Q_h - Q_c)/t_{\text{cycle}} \end{aligned} \quad (21)$$

where the proportionality constant for the investment cost of the heat exchanger, a , is equal to the capital recovery factor times investment cost per unit heat exchanger area and the proportionality constant for the investment cost for the compression and expansion devices, b_2 , is equal to the capital recovery factor times investment cost per unit power input. The initial investment cost is converted to equivalent yearly payment using capital recovery factor [24,25]. The annual energy consumption cost is proportional to the power input, i.e.,

$$C_e = b_3 P = b_3(Q_h - Q_c)/t_{\text{cycle}} \quad (22)$$

where the coefficient, b_3 , is equal to the equivalent annual operation hours corresponding to minimum power input

times price per unit energy [24,25]. Substituting Eqs. (21), (22) into Eq. (20) we have:

$$F = \frac{\dot{Q}_H}{a(A_H + A_L + A_R) + b(Q_H - Q_C)/t_{\text{cycle}}} \quad (23)$$

where $b = b_2 + b_3$. Thus from Eqs. (21)–(23) we have:

$$bF = \frac{[x - a_1(x - 1)]}{k_1 k_2 \left[\frac{x}{k_H(x y - T_{H1})} + \frac{1}{k_L(T_{L1} - y)} + b_1(x - 1) \right] + (x - 1)} \quad (24)$$

where $k_1 = a/b$ and

$$k_2 = \left(\frac{C_f \varepsilon_R}{U_R(1 - \varepsilon_R)} - \frac{C_H}{U_H} \ln(1 - \varepsilon_H) - \frac{C_L}{U_L} \ln(1 - \varepsilon_L) \right).$$

It can be seen from Eq. (24) that bF is a function of two variables x and y [as other parameters are constant for a typical set of operating conditions]. Thus optimizing bF with respect to ' y ' viz. $\frac{\partial bF}{\partial y} = 0$ yields:

$$x(T_{L1} - y) = \sqrt{\frac{k_H}{k_L}} (xy - T_{H1}) \quad (25)$$

Substituting Eq. (25) into Eq. (24) we have:

$$bF = \frac{[x - a_1(x - 1)]}{\left[\frac{x k_5}{(x T_{L1} - T_{H1})} + b_4(x - 1) \right]} \quad (26)$$

where

$$k_3 = \sqrt{\frac{k_H}{k_L}}, \quad k_4 = (1 + k_3) \left(\frac{1}{k_H} + \frac{1}{k_L k_3} \right),$$

$$k_5 = k_1 k_2 k_4 \quad \text{and} \quad b_4 = 1 + k_1 k_2 b_1.$$

Optimizing ' bF ' with respect to x viz. $\frac{\partial bF}{\partial x} = 0$ yields:

$$x_{\text{opt}} = \left\{ b_4 T_{H1} T_{L1} + [T_{H1} T_{L1} [b_4 T_{H1} k_5 (1 - a_1) + b_4 a_1 T_{L1} k_5 - a_1 k_5^2 (1 - a_1)]^{1/2} \right\} \times \{ T_{L1} [b_4 T_{L1} - k_5 (1 - a_1)] \}^{-1} \quad (27)$$

The maximum objective function and the corresponding power input, heating load and COP can be expressed as:

$$bF_{\text{max}} = \frac{[x_{\text{opt}} - a_1(x_{\text{opt}} - 1)]}{\left[\frac{x_{\text{opt}} k_5}{(x_{\text{opt}} T_{L1} - T_{H1})} + b_4(x_{\text{opt}} - 1) \right]} \quad (28)$$

$$P_m = \frac{(x_{\text{opt}} - 1)}{\left[\frac{x_{\text{opt}} k_5}{(x_{\text{opt}} T_{L1} - T_{H1})} + b_1(x_{\text{opt}} - 1) \right]} \quad (29)$$

$$R_{H,m} = \frac{[x_{\text{opt}} - a_1(x_{\text{opt}} - 1)]}{\left[\frac{x_{\text{opt}} k_5}{(x_{\text{opt}} T_{L1} - T_{H1})} + b_1(x_{\text{opt}} - 1) \right]} \quad (30)$$

$$\text{COP}_{H,m} = \frac{[x_{\text{opt}} - a_1(x_{\text{opt}} - 1)]}{(x_{\text{opt}} - 1)} \quad (31)$$

3.1. Special cases

(1) When $\varepsilon_R = 1.0$, i.e., the cycle possesses the condition of perfect regeneration, the maximum thermo-economic function and the corresponding power input, heating load and COP are given by:

$$bF_{\text{max}} = \frac{x_{\text{opt}}}{\left[\frac{x_{\text{opt}} k_5}{(x_{\text{opt}} T_{L1} - T_{H1})} + b_4(x_{\text{opt}} - 1) \right]} \quad (32)$$

$$P_m = \frac{(x_{\text{opt}} - 1)}{\left[\frac{x_{\text{opt}} k_5}{(x_{\text{opt}} T_{L1} - T_{H1})} + b_1(x_{\text{opt}} - 1) \right]} \quad (33)$$

$$R_{Hm} = \frac{x_{\text{opt}}}{\left[\frac{x_{\text{opt}} k_5}{(x_{\text{opt}} T_{L1} - T_{H1})} + b_1(x_{\text{opt}} - 1) \right]} \quad (34)$$

$$\text{COP}_{H,m} = \frac{x_{\text{opt}}}{(x_{\text{opt}} - 1)} \quad (35)$$

where

$$x_{\text{opt}} = \frac{T_{H1} b_4 T_{L1} + \sqrt{b_4 k_5 T_{L1}}}{T_{L1} [b_4 T_{L1} - k_5]} \quad (36)$$

In this case not only the maximum thermo-economic function but also the corresponding heating load of the Stirling cycle is similar to that of an equivalent Carnot cycle. However, for finite regeneration time/area ε_R should be less than unity. It shows that in the investigation of the Stirling cycle it would be difficult in obtaining new conclusions if the regenerator losses were not considered in the analysis.

(2) If $t_R = 0$, i.e., the time of the regenerative processes is negligible as compared with that of the two isothermal processes, the maximum thermo-economic function and the corresponding power input and heating load are:

$$bF_{\text{max}} = \frac{x_{\text{opt}} - a_1(x_{\text{opt}} - 1)}{\left(\frac{k_5 x_{\text{opt}}}{(x_{\text{opt}} T_{L1} - T_{H1})} + (x_{\text{opt}} - 1) \right)} \quad (37)$$

$$P_m = \frac{(x_{\text{opt}} - 1)(x_{\text{opt}} T_{L1} - T_{H1})}{k_5 x_{\text{opt}}} \quad (38)$$

$$R_{Hm} = \frac{[x_{\text{opt}} - a_1(x_{\text{opt}} - 1)](x_{\text{opt}} T_{L1} - T_{H1})}{k_5 x_{\text{opt}}} \quad (39)$$

where

$$x_{\text{opt}} = \left\{ T_{H1} T_{L1} + \sqrt{T_{H1} T_{L1} [T_{H1} k_5 (1 - a_1) + a_1 T_{L1} k_5 - a_1 k_5^2 (1 - a_1)]^{1/2}} \right\} \times \{ T_{L1} [T_{L1} - k_5 (1 - a_1)] \}^{-1} \quad (40)$$

From the above results, we conclude that the Stirling cycle has different performance characteristics for different configurations. Thus, the expressions given by Eqs. (17)–(20) are more general and include all the major irreversibilities associated with this cycle.

(3) When $C_H \rightarrow \infty$ and $C_L \rightarrow \infty$, i.e., the heat capacitance rates of external the reservoirs are infinite, the Stirling heat pump is operating between two infinite heat reservoirs. In such a case, the maximum thermo-economic function and the corresponding power input and heating load are:

$$bF_{\max} = \frac{x_{\text{opt}} - a_1(x_{\text{opt}} - 1)}{\left(\frac{k_5 x_{\text{opt}}}{(x_{\text{opt}} T_L - T_H)} + b_4(x_{\text{opt}} - 1)\right)} \quad (41)$$

$$P_m = \frac{(x_{\text{opt}} - 1)}{\left(\frac{k_5 x_{\text{opt}}}{(x_{\text{opt}} T_L - T_H)} + b_4(x_{\text{opt}} - 1)\right)} \quad (42)$$

$$R_{\text{Hm}} = \frac{[x_{\text{opt}} - a_1(x_{\text{opt}} - 1)]}{\left(\frac{k_5 x_{\text{opt}}}{(x_{\text{opt}} T_L - T_H)} + b_4(x_{\text{opt}} - 1)\right)} \quad (43)$$

where

$$x_{\text{opt}} = \left\{ b_4 T_H T_L + [T_H T_L [b_4 T_H k_5 (1 - a_1) + b_4 a_1 T_L k_5 - a_1 k_5^2 (1 - a_1)]]^{1/2} \right\} \\ \times \{ T_L [b_4 T_L - k_5 (1 - a_1)] \}^{-1} \quad (44)$$

while

$$k_3 = \sqrt{\frac{U_H A_H}{U_L A_L}} \quad \text{and} \quad k_4 = \left(\frac{1}{\sqrt{U_H A_H}} + \frac{1}{\sqrt{U_L A_L}} \right)^2$$

(4) When $t_R = \gamma(t_H + t_L)$, i.e., the time of regeneration is directly proportional to the time of two isothermal processes, the maximum thermo-economic function and the corresponding power input and heating load are:

$$bF_{\max} = \frac{x_{\text{opt}} - a_1(x_{\text{opt}} - 1)}{\left(\frac{(1+\gamma)k_5 x_{\text{opt}}}{(x_{\text{opt}} T_{L1} - T_{H1})} + (x_{\text{opt}} - 1)\right)} \quad (45)$$

$$P_m = \frac{(x_{\text{opt}} - 1)(x_{\text{opt}} T_{L1} - T_{H1})}{(1 + \gamma)k_5 x_{\text{opt}}} \quad (46)$$

$$R_{\text{Hm}} = \frac{[x_{\text{opt}} - a_1(x_{\text{opt}} - 1)](x_{\text{opt}} T_{L1} - T_{H1})}{(1 + \gamma)k_5 x_{\text{opt}}} \quad (47)$$

where

$$x_{\text{opt}} = \left\{ T_{H1} T_{L1} + [T_{H1} T_{L1} [T_{H1} k_6 (1 + \gamma)(1 - a_1) + a_1 T_{L1} k_6 (1 + \gamma) - a_1 k_5^2 (1 + \gamma)(1 - a_1)]]^{1/2} \right\} \\ \times \{ T_{L1} [T_{L1} - k_5 (1 + \gamma)(1 - a_1)] \}^{-1} \quad (48)$$

4. Discussion of results

In order to have the numerical appreciation of the results for the Stirling heat pump, we continue to investigate the effects of the heat sink and heat source temperatures (T_{H1} , T_{L1}), the effectiveness of the heat exchangers (ε_H , ε_L and ε_R), the economic parameter (k_1) and heat capacitance rates (C_H and C_L). While the effect of each one of these parameters is examined and the rest of the parameters are kept constant as ($k_1 = 0.50$, $b_1 = 0.01$, $U_H = U_L = U_R = 1.0$, $\varepsilon_H = \varepsilon_L = \varepsilon_R = 0.75$, $T_{H1} = 330$ K, $T_{L1} = 290$ K, $\lambda = 2.0$, $C_H = C_L = 1.0$ kW/K) and the results obtained are as follows:

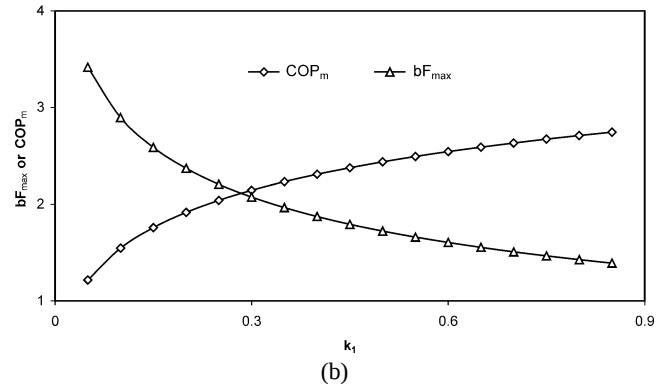
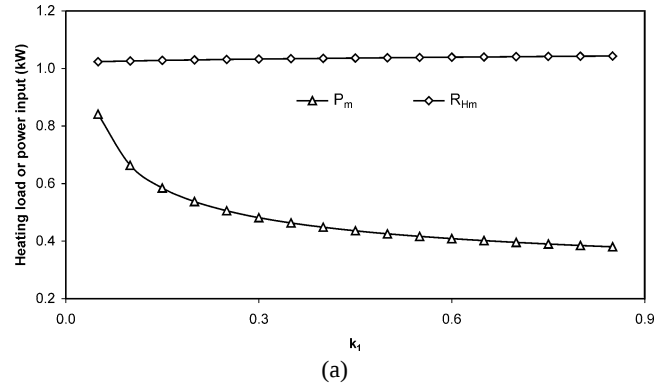


Fig. 2. (a) Heating load and power unput vs k_1 . (b) Objective function and COP vs k_1 .

4.1. Effect of k_1

The effects of economical parameter ($k_1 = a/b$) on the heating load, power input, COP and thermo-economic objective function are shown in Figs. 2(a), (b). It is seen from these figures that the COP increases while the power input and the objective function decrease whereas the change in the heating load is very small as k_1 increases. Since our prime object is to have higher thermo-economic function for a typical set of operating condition. Hence, it is better to have lower value of k_1 for the maximum objective function from the point of view of thermodynamics as well as from the point of view of economics. Again, the effect of k_1 is more pronounced for the maximum objective function and less pronounced for the heating load.

4.2. Effect of effectiveness

The variation of (bF) with the COP for different effectiveness is shown in Fig. 3(a). It is seen that the objective function first increases and then decreases as the COP increases. Thus there is an optimum value of COP at which the objective function attains its maximum. So it would be better to choose the point of maximum objective function in the design of the Stirling heat pump from the point of view of thermodynamics as well as from the point of view of economics.

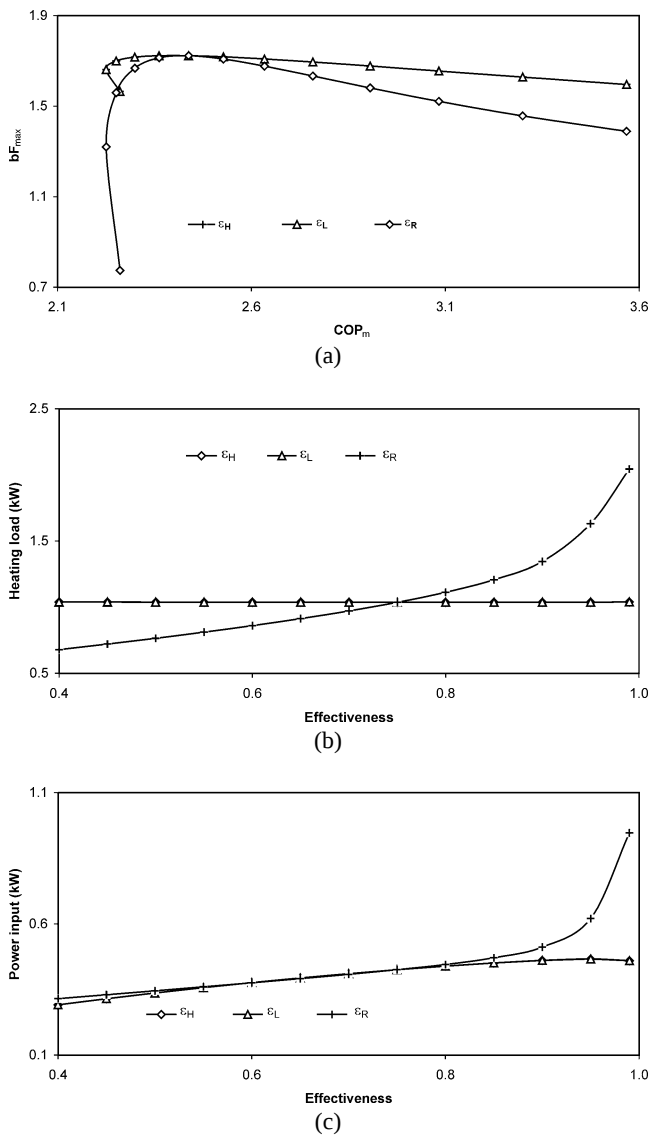


Fig. 3. (a) Objective function vs COP. (b) Heating load vs effectiveness. (c) Power input vs effectiveness.

Figs. 3(b), (c) show the effects of hot- and cold-side and regenerative effectiveness on the heating load and power input for a given set of operating conditions. It is seen from these figures that both the parameters, i.e., the heating load and the power input increase, as the effectiveness on the either side heat exchangers increase. But the change in the heating load and the power input with the hot- and cold-side effectiveness is very small as can be seen from these graphs. So it would be better to select the optimum condition from the point of thermodynamics as well as from the point of view of economics. Again, the effect of regenerative side effectiveness is more pronounced than the hot- and cold-side effectiveness on all the parameters. Also the effects of hot- and cold-side effectiveness are almost the same for these parameters, so the two curves overlap as can be seen from these figures.

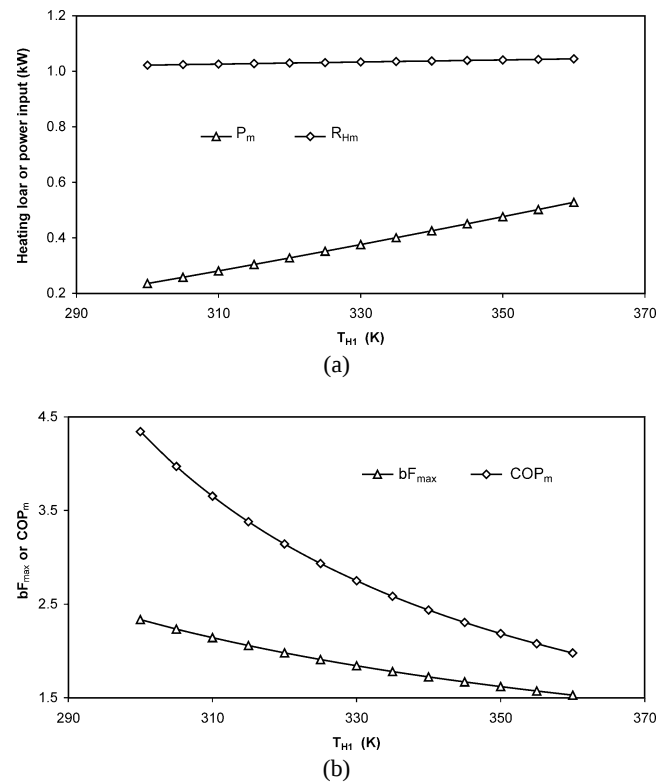


Fig. 4. (a) Heating load and power input vs T_{H1} . (b) Objective function and COP vs T_{H1} .

4.3. Effect of sink and source temperatures

Figs. 4, 5 show the effect of sink and source temperatures on the heating load, COP, power input and the objective function. It is seen from these figures that as the sink side temperature increases, the heating load increases while the COP and the objective function decrease. On the other hand, the heating load decreases while the COP and the objective function increase, as the inlet temperature on the source side reservoir increases. Also there is a small change in the power input in both the cases. Since our prime object is to get higher performance with the lower investment, so it is better to have higher value of T_{L1} and lower value of T_{H1} . Again, the effects of T_{H1} and T_{L1} are more pronounced for the objective function and less pronounced for the heating load.

From the above results we conclude that the sink side inlet temperature should be lower and the source side inlet temperature should be higher for better performance of these cycles. But it is little bit difficult to decrease the sink side temperature rather than to increase the source side inlet temperature and can be done by using low-grade-heat-source such as solar energy [30] or the waste heat [31] from industrial/power plant.

4.4. Effect of heat capacitance rates

The effects of heat capacitance rates (C_H and C_L) on the heating load, power input, COP and the thermo-economic

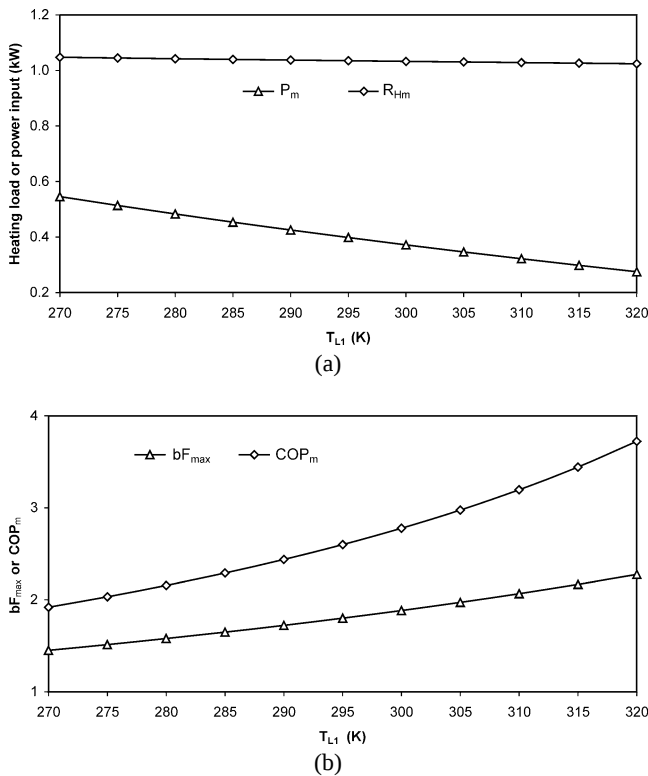


Fig. 5. (a) Heating load and power input vs T_{L1} . (b) Objective function and COP vs T_{L1} .

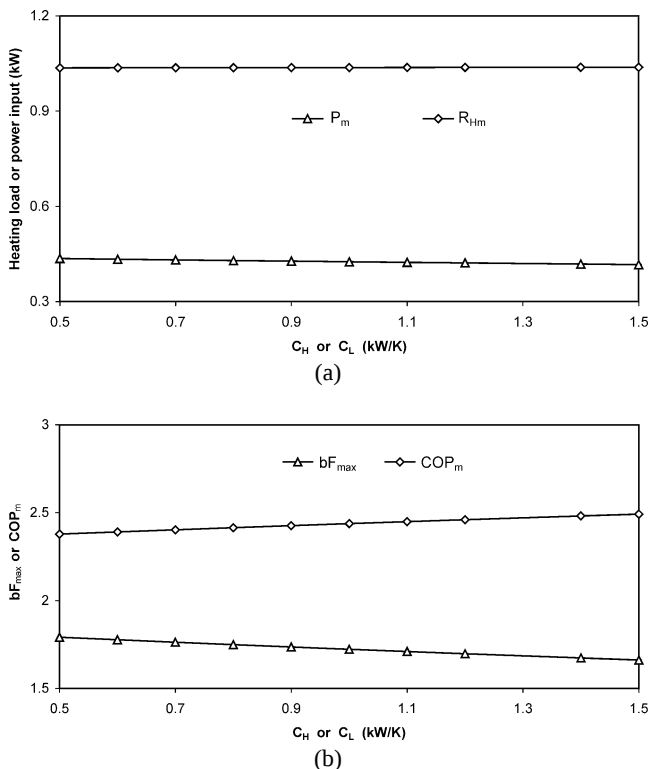


Fig. 6. (a) Heating load and power input vs C_H or C_L . (b) Objective function and COP vs C_H or C_L .

function (bF) are shown in Figs. 6(a), (b). It is seen from these figures that the COP increases while the objective function decreases as the heat capacitance rate on either side heat exchanger increases. On the other hand there is a small change in the power input and heating load, yet the graphs show the trends. Again the effect of C_H and C_L are equal on all the parameters.

Since, the higher value of heat capacitance rate forces the working fluid to absorb/reject heat at lower temperature by decreasing the external irreversibility. Thus, it would be better to have the higher values of heat capacitance rates on both side heat exchangers for better performance from point of view of thermodynamics while it is better to have the lower values of C_H and C_L from economic point of view, which is the prime object of this analysis. Hence, the optimum operating condition should be chosen carefully for better performance from the point of view of thermodynamics as well as from the point of view of economics.

5. Conclusions

The Thermo-economic optimization along with the detailed parametric study of an irreversible Stirling heat pump cycle has been evaluated for the finite heat capacity of external reservoirs. It is found that there is an optimal value of COP for a typical set of operating conditions at which the objective function attains its maximum. The thermo-economic function is found to be increasing function of the source side inlet temperature and the regenerative effectiveness while it is found to be decreasing function of the economic parameter, sink side inlet temperature and heat capacitance rates.

The effects of regenerative side effectiveness and the economic parameter are found to be more than those of the other parameters from the point of view of thermodynamics as well as from the point of view of economics. Moreover, the economic parameter, the source side inlet temperature and the heat capacitance rates can be used as the important criteria in the design of an irreversible Stirling heat pump cycle and other similar cycles too. Again, it is also found that the inlet temperature on the source side reservoir should be little bit higher for better performance of the cycle. It can be done by using low grade energy like solar energy and/or the waste heat from the industrial and power plants for the overall better performance of this cycle.

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